THE TEMPERATURE OF AN ADIABATIC SURFACE AT A TURBULENT BOUNDARY LAYER

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A calculation of the temperature decrease of an adiabatic surface at a supersonic turbulent boundary layer is conducted. It is shown that the temperature decrease is a consequence of the appearance of a vortex chain in the flow near the walls. Comparison of calculated data with experimental gives qualitative agreement.

In studies of supersonic flow over surfaces with rectangular openings, a decrease in recovery temperature below the temperature of a surface with laminar boundary layer has been observed [1, 2]. The temperature decrease noted in the region of current juncture contradicts established concepts, according to which the turbulizing role of the depression should obviously lead to a temperature increase.

Measurement of surface temperatures over the openings gave the results shown in Fig. 1. Measurements were conducted in a current with Mach number M = 1.69 in a flat channel, with Reynolds number Re $= 4.4 \cdot 10^5$ at the point of discontinuity. The results are presented in the form of the functional dependence of the ratio of the recovery coefficient r to the corresponding recovery coefficient r_0 on a smooth plate on length L, measured in units of h_0. The measuring element, consisting of a copper insert, 3 mm in diameter, with thermocouple, was located at point *a*. As is evident, with increase in L the wall temperature beyond the opening first decreases, and then rises smoothly, having a clearly defined minimum in the interval $L/h_0 \approx 2$-4.

In [3], in an examination of high velocity air flow around cylinders, the suggestion was made that the temperature decrease is produced by a well defined unstable flow, arising through a regular breakaway and departure downward in the flow of large scale vortices. Following the proposition of that work, we will attempt to conduct an analysis of the cooling observed with a vortex boundary layer, considering the instability present in this case.

We will assume that the vortex boundary layer (Fig. 2) consists of a thin dissipative flow region 2 and an external isoentropic region, the flow within which, excluding the vortex centers 1, may be regarded as potential. The velocity of the incident flow is equal to V_∞, and the vortex center translation velocity is V_v. The vortex spacing and distance from the vortex center to the boundary with the viscous sublayer will be designated by l_v and h, respectively. We will then find the temperature of surface 3, and compare the same with the temperature of an adiabatic surface with stable flow.

We will introduce the values of velocity v and thermodynamic temperature T at the external boundary of the dissipative flow region. Then the recovery temperature of the surface flowed over will be determined by

$$T_w = T + r\,\frac{v^2}{2c_p}, \tag{1}$$

where c_p is the heat capacity of the gas at constant pressure.

The conditions at the external boundary of the dissipative flow region are found from the LaGrange equation for planar unstable potential flow, which has the form

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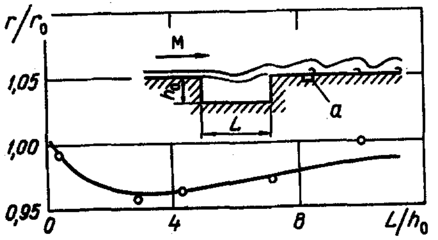

Fig.1

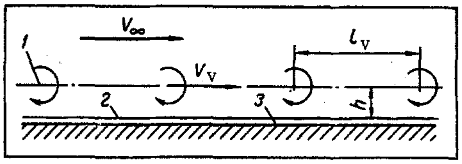

Fig.2

Fig.1. Measurement of recovery coefficient in the region of vortex boundary layer depending on L for M = 1.69.

Fig.2. Scheme of flow: 1) centers of vortices; 2) viscous sublayer; 3) wall.

$$\frac{\partial \varphi}{\partial \tau}+\frac{v^2}{2}+c_pT=\frac{V_\infty^2}{2}+c_pT_\infty=\text{const},\tag{2}$$

where φ is the velocity potential, and T_∞ is the temperature of the undisturbed flow. The constant here is the enthalpy of friction. Equation (2) is obtained from the integral of the LaGrange equation for motion by means of the isoentropic dependences between the gas parameters, and an examination of the region far above the flow, where the velocity is constant. The local velocity v can be depicted as the difference between the undisturbed flow velocity, and the velocity u, introduced in this case by the vortex system, i.e.,

$$v=V_\infty-u.\tag{3}$$

After simple computations, assuming the vortex intensity to be small, and consequently, u much smaller than V_∞, we obtain the formula for wall temperature

$$T_w=T_\infty+r\frac{V_\infty^2}{2c_p}-\frac{1}{c_p}\frac{\partial\varphi}{\partial\tau}-\frac{(1-r)}{c_p}V_\infty u.\tag{4}$$

Inasmuch as in a vortex boundary layer the vortex passage frequency is of the order of 10^4 Hz, it is sufficiently valid to assume that the sensor employed registered only an average temperature, equal to

$$\overline{T}_w=\frac{1}{\tau}\int_0^\tau T_w d\tau=T_\infty+r\frac{V_\infty^2}{2c_p}-\frac{1}{c_p\tau}\int_0^\tau\frac{\partial\varphi}{\partial\tau}\partial\tau+\frac{(1-r)}{c_p\tau}V_\infty\int_0^\tau u d\tau.\tag{5}$$

For a fixed point φ will be a function of τ only, hence

$$\frac{1}{\tau}\int\frac{\partial\varphi}{\partial\tau}d\tau=\frac{\varphi_\tau-\varphi_0}{\tau}.$$

We will introduce an expression for the average induced velocity

$$\frac{1}{\tau}\int_0^\tau u d\tau=\overline{u}.$$

Then the departure in wall temperature for the case examined from wall temperature with stable flow will be determined by

$$\Delta T=\left(T_\infty+r\frac{V_\infty^2}{2c_p}\right)-\overline{T}_w=\frac{1}{c_p}\frac{\varphi_\tau-\varphi_0}{\tau}-\frac{1-r}{c_p}V_\infty\overline{u}.\tag{6}$$

The difference in potentials is found by examination of the translation of a relatively stationary point on the wall n of a pair of vortices, consisting of a real and mirror vortex, with intensity $\varkappa$ equal in magnitude, but opposite in sign. For a sufficiently long time period we have

$$\frac{\varphi_\tau-\varphi_0}{\tau}=2\pi n\varkappa,\tag{7}$$

where n is the vortex passage frequency.

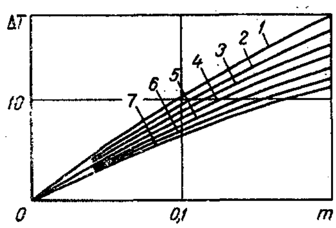

Fig.3

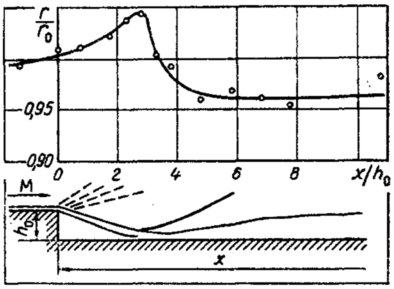

Fig.4

Fig.3. Temperature decrease for an adiabatic surface for $l^* = 28$, $r = 0.85$: 1) $V_\infty = 560$ m/sec; 2) 540; 3) 520; 4) 500; 5) 480; 6) 460; 7) 440.

Fig.4. Recovery coefficient for flow over a backward-facing step.

In order to determine the induced velocity, we shall use Eq. (3), which could also be interpreted from the viewpoint of current superposition. Assuming the value of u to be much lower than V_∞, one can go further and assume it to be much lower than the velocity of sound. This then permits the use of the theory of the functions of a complex variable in calculating u.

Calculations by well-known formulae for instantaneous velocity values induced on the center line of the vortex path, with symmetric distribution of vortices, indicate strong oscillations over time in the values of these velocities, so that obtaining an average induced velocity is coupled with certain difficulties. For simplification, if we take as the average velocity the velocity induced by a vortex layer with intensity of evenly distributed vortices (linear circulation) $\varkappa_c = \varkappa/l_v$, we will have

$$\bar{u} = 2\pi\varkappa/l_v. \tag{8}$$

Further, introducing the relative vortex translation velocity $b = V_v/V_\infty$, and determining thereby the vortex passage frequency $n = bV_\infty/l_v$, substitution in Eq. (6) will produce

$$\Delta T = \frac{V_\infty \bar{u}}{c_p}(b + r - 1). \tag{9}$$

As a preliminary hypothesis, we will assume the region of dissipative flow to be a laminar boundary layer, beginning at the point of flow union. Then the value of the recovery coefficient $r \approx 0.85$. Experiments show that the relative vortex translation velocity is unstable, but on the average its value may be regarded as equal to $b \approx 0.85$. Thus, ΔT determines a temperature decrease (cooling) of the surface when a vortex chain appears above it.

From Eq. (9) it is evident that in determination of cooling, a knowledge of $\bar{u}$ is insufficient, this quantity also being determined by the unknown vortex intensity $\varkappa$.

In order to find $\bar{u}$, we will examine the question of the vortex translation velocity relative to the surrounding medium. Returning to the vortex path with symmetrical vortex distribution, it may be noted that, inasmuch as no one vortex chain induces a velocity upon itself, the chain translates only under the influence of a vortex of another row. Under such conditions the vector chain velocity is found by the well known formula obtained through the theory of the functions of a complex variable,

$$V_v^0 = \frac{\pi\varkappa}{l_v}\,\mathrm{cth}\left(\frac{2\pi h}{l_v}\right). \tag{10}$$

We note that an analogous system of flow was examined in [4]. However, the calculation conducted therein, with the assumption $V_v = V_\infty$ cannot be considered valid, since an assumption of that nature is analogous to taking $V_v^0 = 0$, which deprives the problem of its content, inasmuch as a nonzero value for the relative vortex velocity in Eq. (10) is a necessary condition for the existence of vortices.

Employing Eq. (10), after a series of simple computations, we obtain the expression for the induced velocity

$$\bar{u} = 2(1-b)V_{\infty}\,\mathrm{th}\left(\frac{2\pi h}{l_{B}}\right). \tag{11}$$

We may now verify the degree of validity of the assumption that $\bar{u}$ is small in comparison with V_{∞}. For an approximate evaluation, we will take some median experimental data: b = 0.85 and $l_V/h = 28$. Then $\bar{u}/V_{\infty} \approx 0.07$, which supports the assumption made.

Introducing the simplified notation (1−b) = m and $l_V/h = l^*$, we obtain the final expression for temperature decrease

$$\Delta T = \frac{2V_{\infty}}{c_p}\,[m(r-m)]\,\mathrm{th}\left(\frac{2\pi}{l^*}\right). \tag{12}$$

Calculations for experiments conducted are presented in Fig. 3, where the ΔT values are given in °C. From the distribution of the curves calculated for different incident flow velocities, it is evident that with increase in V_{∞}, the degree of "cooling" of the plate surface increases, other conditions being equal, with an increase in intensity of the vortices comprising the vortex path.

The approximate character of Eq. (12) should be noted. This approximateness is due not only to assuming isoentropic flow in the external regions of the near-wall layer, but also to taking the quantity l^* as constant, since it in fact increases with movement down the flow. The assumption of an infinite vortex chain is also an idealization. (One might also note the sufficiently close analogy to Karman's calculation scheme for the vortex path, in which the ratio between vortex spacing and vortex width is taken as constant for ease in calculation, when in fact pictures of such paths in [5] show that this is not the case.) Moreover, the attainment of accurate values of temperature decrease from Eq. (12) is also impossible due to the lack of accurate values for m and l^*. It is most difficult to determine l^* due to the great difficulty of determining h, wherein it is necessary to consider the thickness of the viscous sublayer. To all that has been said, it might be added that the calculation does not consider viscous deformation of the vortices, and uses an approximate mean value for the induced velocity.

The scheme of flow examined indicates a necessary rise in recovery temperature as distance from the back edge of the depression increases, as a consequence of a decrease in velocity of relative vortex motion with a decrease in vortex intensity, these effects being produced by viscosity. Despite the approximate nature of the calculation, the results obtained in Fig. 3 give values which coincide to within one order with experimental results. Thus, the temperature decrease beyond the depression in the experiments described for L ≈ 3h (see Fig. 1) was ΔT ≈ 4-5°C.

The rise of a vortex boundary layer (although there are no direct indications for this) may also explain the temperature decrease of a nonconductive surface beyond a theoretical plate, from which flows a supersonic current, as is evident from the example of flow of the second type, presented in [1], and shown experimentally in Fig. 4, obtained for M = 1.7.

In conclusion, it can be said that the proposal of a vortex boundary layer consisting of a viscous sublayer with unstable motion on the exterior boundary and a nonviscous region with potential flow, excluding the concentrated vortices, is evidently justifiable. In any case it does give a qualitatively satisfactory explanation of the temperature decrease mechanism.

NOTATION

V_{∞}	is the incident flow velocity;
V_V	is the vortex velocity;
v	is the local velocity;
u	is the induced velocity;
T	is the thermodynamic temperature;
T_W, T_{∞}	are the recovery temperature and undisturbed flow temperature, respectively;
L	is the length of depression;
h_0	is the depth of depression;
h	is the distance from vortex center to wall;
b	is the relative vortex velocity;
l_V	is the vortex spacing;
r	is the recovery coefficient;
r_0	is the recovery coefficient on smooth surface;

c_p is the gas heat capacity at constant pressure;
n is the vortex passage frequency;
Re is the Reynolds number;
M is the Mach number;
φ is the velocity potential;
τ is the time;
$\varkappa$ is the vortex intensity.

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